[Final manuscript. Forthcoming in *HOPOS*]

Sean Morris. *Quine, New Foundations, and the Philosophy of Set Theory*. Cambridge: Cambridge University Press, 2018. Pp. x + 209. $105.00 (hbk), ISBN 9781107152502.

Sander Verhaegh
TiLPS, Tilburg University

This is an important book about W. V. Quine's contributions to set theory, a largely neglected component of his philosophy. Today, Quine is best known for his naturalism, for his indeterminacy theses, and for his views about analyticity, meaning, and ontology. When Quine was asked what *he himself* regarded as his most important philosophical contributions, however, he listed New Foundations (NF) as one of his two most significant achievements.[1] Morris' book is a successful attempt to do justice to Quine's axiomatic set theory. He analyzes its most important mathematical properties, he reconstructs Quine's motives for developing the system, and he compares it to alternative axiomatizations like Type Theory and Zermelo-Fraenkel set theory (ZF). Although Morris makes it clear from the outset that it is not his aim "to argue for NF as the *correct* set theory" (p. 7, my emphasis), he does argue that Quine's system has interesting mathematical properties, such as allowing for absolute complements, making mathematical sense of Cantor's absolute infinities, and allowing 'big sets' like the set of ordinal numbers, the set of cardinal numbers, and the universal set.

Yet Morris does not restrict himself to a *mathematical* analysis of NF. First and foremost, he aims (1) to use NF as an illustration of Quine's *philosophy* of set theory and (2) to apply this philosophy to present-day discussions about sets. For, according to Morris, contemporary debates about set theory are often guided by the mistaken assumption that the goal of set theory is to uncover "the essence of sethood" (p. 186). Many defenders of the iterative conception of set (most notably, George Boolos and Hao Wang), for example, view set theory as an enterprise in which we try to uncover what sets are ultimately like. Morris, on the other hand, argues that set theory should not aim to offer a *conceptual analysis* of the notion of set. Rather, we should try to *explicate* the concept and find out which set theory best serves our mathematical purposes. Appealing to Quine's seminal discussion of explication in *Word and Object*, Morris argues that we should not assume that there is only *one* viable conception of set. Just as there is not one correct account of what the ordered pair is beyond the demand that any explication should do justice to the principle 'if $\langle x, y\rangle = \langle z, w\rangle$, then $x = z$ and $y = w$', there is no correct account of what a set is beyond the question of whether a particular system fulfills the aims of set theory, such as unifying the "diverse bodies of mathematics into a single framework" (p. 60) and offering "a mathematically interesting account of the infinite" (p. 172).

Morris' book is divided into three parts. Part I offers a detailed and illuminating history of set theory from Cantor to Quine. Morris not only outlines the familiar story of how set theory gradually emerged in the work of Georg Cantor, how Bertrand Russell's discovery of the paradoxes led to a foundational crisis, and how mathematicians like Zermelo

[1] See "Un futuro plausibile secondo Willard Quine, il più importante pensatore d'America Abbasso i becchini della filosofia". Interview by Ermanno Bencivenga. *La Stampa*, August 12, 1998. The second important contribution Quine mentions is *Word and Object*.

subsequently tried to develop consistent set theories; he also shows that the main participants in these early debates were all partly guided by the pragmatic approach favored by Morris. Cantor, for example, regarded all available definitions of the real numbers as equally viable as long as they all yielded "the mathematical properties required of the real numbers" (p. 20), whereas Zermelo regarded set theory as a "largely pragmatic endeavor" after the discovery of the paradoxes. Zermelo's goal was not to offer an intuitive set theory but to "restrict the notion of set sufficiently to exclude contradiction but to maintain enough of the theory that it can continue its foundational purpose for all mathematics" (p. 49).

Still, Morris maintains that "Quine was really the first" to develop this pragmatic conception into "an explicit philosophical approach to set theory" (p. 85). Part II of his book is devoted to spelling out the details of this approach and to reconstruct its development. Morris convincingly argues that Quine adopted the approach from Russell as a graduate student, when he tried to rework the first 500 pages of Whitehead and Russell's *Principia Mathematica.* Quine quickly started to out-Russell Russell, however, when he realized that many of the latter's choices were motivated by philosophical rather than mathematical reasons. Although Russell's restriction of the set-theoretic universe is an attempt to avoid the set-theoretic paradoxes, for example, his more specific demand that all the members of a class are alike with respect to type (such that if an individual is a member of a class x, then x must be composed exclusively of individuals) is an "ontological", and hence an illegitimate philosophical restriction. Part II also aims to answer the question why Quine advocates a "largely pluralistic" (p. 125) approach to set theory, while being generally sceptical about Carnap's principle of tolerance. Why doesn't Quine accept that we are (fallibly) committed to one axiomatic system (such as ZF) as our best current theory, just as he takes us to be (again, fallibly) committed to classical first-order quantification theory in logic? Morris convincingly answers that there is no tension here as Quine views set theory as an unsettled science, such that it still make sense to adopt "an exploratory and experimental approach to the subject" (p. 134). Elementary logic, on the other hand, is viewed by Quine as "a largely settled science, fully integrated into our current best theory" (p. 143).

Part III, finally, applies Quine's pragmatic approach to present-day set theory. Contemporary debates about sets are strongly guided by Boolos' discussion of the iterative conception as expressed by ZF. In his seminal paper "The Iterative Conception of Set" (1971), Boolos argued that is ZF is an "independently motivated theory of sets" because it ties in with our intuitions about the nature of sets. The intuition behind the iterative conception is that a set is "a collection of definite elements of our thought". We begin with individuals, form a collection of these individuals at stage zero, and repeat the procedure at every subsequent stage. An appealing feature of this conception, Boolos argues, is that it is impossible to construct sets that are members of themselves. After all, if we consider sets to be collections of definite elements of our thought, we "don't suppose that what we come up with" in collecting these elements could be "one of the very things we combined" (1971, 17-8). Morris, however, argues that our intuitions about sets are not to be trusted. Not only have the paradoxes shown us that set theory is inconsistent in its most intuitive version, Morris also argues that the intuitions like the ones Boolos appeals to should be used with great caution:

> As [Boolos] presents it […], set theory sounds like a theory of collections of very ordinary physical objects: Here are some things, and we collect them together. This sounds fine when we think about, say, rocks or paper clips, but […] it would hardly make sense of abstract collections of numbers or, say, of sets themselves […] What made set theory so important was its ability to make sense of the infinite […] How

> would our collecting together an infinite number of objects be made sense of according to our ability to bind objects together? (p. 159-60)

Most importantly, however, Morris argues that the iterative conception fails to do what a set theory ought to do. For the iterative conception itself does not deliver an adequate set theory. We need to adopt an additional axiom like Replacement, for example, to obtain a satisfactory theory about infinite numbers. Similarly, the Axiom of Extensionality—a principle that in Boolos' view "enjoys a special epistemological status shared by none of the other axioms" (1971, 27)—does not follow from the iterative conception either. The intuitive approach, in other words, "only gets us so far, mathematically speaking" (p. 164). In the end, Morris concludes, we will always need to return to the question whether or not a particular set theory serves our mathematical needs.

All in all, Morris' book offers a subtle but strong argument for a reappraisal of Quine's New Foundations, arguing that the system has interesting, though often neglected, mathematical properties. First and foremost, however, Morris' book is a convincing defense of Quine's *approach* to set theory, showing that the latter's naturalism is not only an abstract theory about the nature of inquiry but also a philosophy that has important and concrete implications for fundamental debates in the sciences.

Literature

Boolos, G. (1971). The Iterative Conception of Sets. In Jeffrey, R. (1998, ed.). *Logic, Logic, and Logic* (pp. 13-29). Cambridge, MA: Harvard University Press.